Solving Systems of Equations Algebraically

In order to solve for two variables, you need to have two equations. If you only have one equation there are an infinite amount of **ordered pairs (x,y)** that will work. For example:

4x – 2y = 16 you can have x = 4 and y = 0 (4,0) and (2, -2) and (0, -4) and an infinite amount of others. To be able to solve for a single ordered pair, you need a second equation.

When we introduce the second equation, we will be able to solve for a single ordered pair that will work in both equations. There are two ways to solve a system of equations (algebraically and graphically). We will focus on solving algebraically. There are two methods of solving algebraically (substitution and elimination). The key to both of them is changing one (or both) equations so there is only one variable to solve for. Then you follow all the rules of solving for the one variable. Then plug the value back into one of the original equations to find the value of the second variable. Always state your answer as an ordered pair.

SUBSTITUTION

Example: x = 3y + 8
5x + 2y = 6

Substitute 3y +8 for the x in the 2nd equation

5(3y + 8) + 2y = 6

Distribute and solve:

15y + 40 + 2y = 6

17y + 40 = 6

$\frac{17y}{17} = \frac{-34}{17}$

y = -2

substitute the value for y back in to find x.

y = -2

x = 3(-2) + 8

x= -6 + 8

x = 2

(2, -2) *State answer as an ordered pair (x,y)*

Check in BOTH ORIGINAL EQUATIONS!

x = 3y + 8

(2) = 3(-2) + 8

2 = -6 + 8

2 = 2 ✓

5x + 2y = 6

5(2) + 2(-2) = 6

10 – 6 = 4

4 = 4 ✓

Solve each system and check (in both equations):

a) x = 2y + 1
5x – 6y = 13

b) $y = 3x + 4$
$9x + 2y = -37$

c) $4x + 2y = 24$
$10x + y = 8$

d) $6x - 5y = 20$
$x + 3y = 11$

e) $7x + 9y = -74$
$4x + y = -5$

f) $8x + 3y = 35$
$10x - y = 1$

Mixed Problems

1) $y = 3x - 8$
   $6x - 5y = 31$

2) $x = 2y + 9$
   $5x + 8y = 117$

3) $y = 2x - 3$
$6x - 5y = 31$

4) $4x + y = -9$
$12x - 7y = 123$

5) $x - 7y = 19$
$6x + 11y = 61$

6) $3y - 2 = x$
$3x - 7y = -16$

Answer Key: ***(1,-5) 2) (17,4) 3) -4,-11) 4) (1.5, -15) 5) (12,-1) 6) (-17,-5)***

Solving Systems with Linear Combinations ("Elimination"):

Sometimes solving a system of equations using substitution can be very difficult. For these problems we solve using Linear Combinations (or Elimination). With elimination you solve by eliminating one of the variables. This is accomplished by adding the 2 equations together. Before you can add the equations together, you need one of the two variables to have two things:

1) Same Coefficient
2) Different Signs (one positive and one negative)

When you add terms with the same coefficient and different signs, the term drops out. You then solve for the variable that is left. After you have solved for one variable, you plug the value into one of the original equations and solve for the 2nd variable (just like Substitution). Then, you check the solution in both original equations. The only difference between Substitution and Elimination is how you solve for the 1st variable. After that they are the same.

Examples:
A) Sometimes it works out that the 2 equations already have a variable with the same coefficient and different signs. You can then just add the equations:

$3x + 4y = 10$ ***(The +4y and -4y cancel out***
$5x - 4y = -58$ ***leaving you with just 8x.)***
$\frac{8x}{8} = \frac{-48}{8}$

$x = -6$

Plug x = -6 in:
$3(-6) + 4y = 10$
$-18 + 4y = 10$
$+18 \quad +18$
$\frac{4y}{4} = \frac{28}{4}$
$y = 7$

3x+4y = 10
$3(-6) + 4(7) = 10$
$-18 + 28 = 10$
$10 = 10$(*check*)
5x – 4y = -58
$5(-6) - 4(7) = -58$
$-30 - 28 = -58$
$-58 = -58$(*check*)

Final Solution: (-6, 7) CHECK IN BOTH!!!!

B) Sometimes (usually) the equations do not have same coefficient and different signs, so we have a little bit of manipulating to do.

$3x + 8y = 25$ ***With this system, nothing will drop out if we just add the***
$5x + 4y = 23$ ***equations. So we will multiply the bottom one by (-2).***
$-2(5x + 4y = 23)$ ***Now the y's have the same coefficient with different signs.***
$-10x - 8y = -46$
$3x + 8y = 25$
$-10x - 8y = -46$
$\frac{-7x}{-7} = \frac{-21}{-7}$

$x = 3$

Now plug x = 3 in:
$3(3) + 8y = 25$
$9 + 8y = 25$
$-9 \quad -9$
$\frac{8y}{8} = \frac{16}{8}$
$y = 2$

3x + 8y = 25
$3(3)+8(2) = 25$
$9 + 16 = 25$
$25 = 25$ (*check*)
5x + 4y = 23
$5(3) + 4(2) = 23$
$15 + 8 = 23$
$23 = 23$ (*check*)

Final Solution: (3,2) CHECK IN BOTH!!!!

C. Sometimes we need to manipulate both equations. We can do this by

"criss crossing the coefficients."

6x + 7y = 11
5x – 6y = -50

This is different than Example B, because no coeffcient goes into another evenly.

You need the negative sign to change the 6x to negative so the signs will be different.

-5(6x + 7y = 11)
6(5x – 6y = -50)

You can also use 5 and -6.

You can also "criss cross" the y coefficients.

-30x – 35y = -55
30x – 36y = -300
- 71y = -355
-71 -71

y = 5

Plug in y = 5
5x – 6(5) = -50
5x – 30 = -50
+30 +30
5x = -20
5 5

x = -4

6x + 7y = 11
6(-4) + 7(5) = 11
-24 + 35 = 11
11 = 11 (*check*)

5x – 6y = -50
5(-4) – 6(5) = -50
-20 – 30 = -50
-50 = -50 (*check*)

Final Solution: (-4, 5) CHECK IN BOTH!!!!

Practice:

1) 7x + 3y = 10
5x – 6y = 56

2) $11x + 5y = 27$
$4x + 6y = 60$

3) $9x + 7y = 126$
$7x – 9y = -32$

4) $12x - 5y = 63$
$8x + 3y = 23$

5) $5x + 9y = 14$
$6x + 11y = 18$

6) $10x - 9y = 36$
$4x + 3y = -12$

7) $5x + 6y = 42$
$3x + 14y = 20$

8) $7x - 5y = -42$
$8x + 3y = -48$

9) $4x - 3y = 19$
$8x + 5y = 159$

Mixed Substitution and Elimination:

Solve each system algebraically:

1) $5x - 2y = -9$
$7x + 2y = -27$

2) $-4x + 2y = -16$
$5x - 3y = 19$

3) $x = 2y - 6$
$5y - 3x = 11$

4) $5x - 6y = -74$
$7x + 5y = 17$

5) $4x - 5 = y$
$7x + 5y = 83$

6) $7x + 4y = -11$
$5x + 2y = -13$

7) $5x - 6y = -17$
$3x + 8y = -16$

8) $x = 6 + 2y$
$6x - 5y = 15$

9) $6x + 5y = 23$
$11x + 4y = 6$

10) $y = 3x + 4$
$8x - 9y = 59$

11) $12x - 7y = 46$
$4x + 3y = -6$

12) $9x - 4y = -88$
$2x + 5y = 4$

13) $24x - 6y = -66$
$12x - 3y = -33$

14) $5x - 6y = 42$
$15x - 18y = 54$

15) $7x + 6y = -12$
$5x + 2y = -20$

16) $13x - 3y = 78$
$4x + 6y = -66$

17) $2y - 5 = x$
$4x - 11y = -38$

18) $3x - 7y = -10$
$5x + 12y = -64$

19) $6x - 17y = -104$
$4x - 7y = -39$

20) $9x - 5y = -43$
$3x + 11y = 87$

21) $9x = 11y + 25$
$5x - 12y = 8$

22) $6y = 5x - 38$
$7x + 9y = 1$

23) $6x + 5y = 33$
$5x + 37 = 3y$

24) $y = 3x + 5$
$12x - 7y = 1$

Answer Key to Algebraic Systems:

1) (-3,-3)	7) (-4, -.5)	13) many sol.	19) (2.5, 7)
2) (5,2)	8) (0,-3)	14) no sol.	20) (-1/3, 8)
3) (8,7)	9) (-2,7)	15) (-6,5)	21) (4,1)
4) (-4,9)	10) (-5,-11)	16) (3,-13)	22) (4,-3)
5) (4,11)	11) (1.5, -4)	17) (7,6)	23) (-2,9)
6) (-5,6)	12)(-8, 4)	18) (-8, -2)	24) (-4,-7)

Extra Practice (do in NB)

1) $6x - 5y = -7$
$11x + 5y = 58$

2) $5x + 4y = -69$
$5x - 7y = 52$

3) $6x + 7y = -28$
$5x - 14y = -182$

4) $11x - 4y = 53$
$7x - 8y = 1$

5) $3x - 7y = 42$
$2x + 5y = 57$

6) $9x - 4y = 177$
$6x - 5y = 111$

7) $8x - 11y = 77$
$6x + 4y = -28$

8) $13x - 2y = 72$
$9x + 5y = -14$

9) $12x = 20 - 8y$
$5x - 6y = -57$

10) $5y = 8x + 97$
$10x + 7y = 51$

Answer Key:

1) (3, 5)

2) (-5, -11)

3) (-14, 8)

4) (7,6)

5) (21,3)

6) (21, 3)

7) (0, -7)

8) (4, -10)

9) (-3,7)

10) (-4, 13)

Systems Review

1) $7x - 4y = -86$
$9x - 4y = -98$

2) $3x - 10y = -18$
$9x + 8y = -168$

3) $5x + 8y = 70$
$-4x - 5y = -56$

4) $10x + 11y = 37$
$8x - 7y = -160$

5) $6x + 13y = -66$
$4x + 7y = -34$

6) $5x - 9y = 22$
$8x - 5y = 101$

Answer Key:
1) (-6,11) 2) (-16,-3) 3) (14,0) 4) (-9.5, 12) 5) (2,-6) 6) (17,7)

Mixed Word Problems

1) After a big Yankee win, Didi bought 4 slices of pizza and 2 cokes for $10.20 and Giancarlo bought 3 slices of pizza and 3 cokes for $9.90. Find the price of one coke. Find the price of 1 slice of pizza.

Let:
X = price of 1 slice of pizza
Y = price of 1 coke

(You can use other letters for the variables, like P for pizza or C for coke. I just LOVE X and Y!!)

Solve:

1st Variable:

$4x + 2y = \$10.20$
$3x + 3y = \$9.90$

$-3(4x + 2y = \$10.20)$
$4(3x + 3y = \$9.90)$

$-12x - 6y = -\$30.60$
$12x + 12y = \$39.60$

$\frac{6y}{6} = \frac{\$9.00}{6}$

$Y = \$1.50$

2nd Variable

$y = \$1.50$

$4x + 2y = \$10.20$
$4x + 2(\$1.50) = \$10.20$
$4x + \$3.00 = \$10.20$
$-\$3.00 = -\$3.00$

$\frac{4x}{4} = \frac{\$7.20}{4}$

$x = \$1.80$

Check:

$4x + 2y = \$10.20$
$4(\$1.80)+2(\$1.50)=\$10.20$
$\$7.20 + \$3.00 = \$10.20$
$\$10.20 = \$10.20$

$3x + 4y = \$9.90$
$3(\$1.80)+\$3(\$1.50)=\$9.90$
$\$5.40 + \$4.50 = \$9.90$
$\$9.90 = \$9.90$

The price of 1 coke is $1.50 and the price of 1 slice of pizza is $1.80

****You always want to state your answer to a word problem with a sentence. You DO NOT make your final answer an ordered pair. In the previous sections we always stated our answer as an ordered pair because the question WAS THE SYSTEM OF EQUATIONS. But with the word problems, the system exists only because we created it as a way to get the answers asked for in the question.***

2) Brett went to the donut shop and bought 6 donuts and 4 large coffees for $8.92. Chase went in right after Brett and bought 5 doughnuts and 6 large coffees for $10.50. Find the price of 1 large coffee. Find the price of 1 donut. Gary went in and bought 3 donuts and 2 large coffees. How much did he pay?

3) Greg and CC went to the burger stand and bought dinner. Greg had 2 cheeseburgers and 3 fries. CC bought 3 cheeseburgers and 2 fries. Greg paid $17.45. CC paid $16.55. How much would 2 cheeseburgers and 1 fries cost?

4) Aaron and Masahiro went shopping for some new Yankee gear. Aaron bought 4 sweatshirts and 5 t-shirts for $254. Masahiro bought 2 sweatshirts and 4 t-shirts for $154. How much would 2 sweatshirts and 3 t-shirts cost?

5) Maggie and Erin went to see *Frozen* and went to the snack bar before finding their seats. Maggie paid $11.05 for 2 candy bars and 3 sodas. Erin paid $17.55 for 3 candy bars and 5 sodas. Find the total cost of 4 candy bars and 1 soda.

6) Sam and Peter went to the pizzeria and ordered some slices. Sam bought 2 slices of Sicilian and 2 regular and his bill was $10. Peter bought 3 slices of Sicilian and 1 regular for $10.50. How much would 4 Sicilian and 5 slices of regular cost?

7) Solid ties cost $21 and striped ties cost $24. The store sold 200 ties and made $4,413. How many of each were sold?

8) At a movie theater adult tickets cost $9.00 and child tickets cost $4.00. 120 people attended the last showing of *Silver Linings Playbook* and $720 was collected at the ticket booth. How many of each ticket was sold?

9) A jar of change was filled with only quarters and dimes. If there were 600 coins in the jar and there was $121.05 in the jar, how many of each coin were there?

10) A 35-minute phone call cost $4.95. Introductory minutes cost $.16/min and additional minutes are $.11/min. How many minutes were billed at each rate?

11) A 32-minute phone call cost $3.01. Introductory minutes cost $.17/min and additional minutes are $.08/min. How many minutes were billed at each rate?

12) There was a jar of coins filled only with nickels and quarters. If there is $53.00 in the jar and there is a total of 300 coins, how many of each coin are in the jar?

Answer Key

1) 1 coke cost $1.50, 1 slice cost $1.80.
2) 1 large coffee cost $1.15, 1 donut cost $.72. Gary paid $4.46
3) 2 Cheeseburgers and 1 Fries cost $9.45. (CB cost $2.95 and fries cost $3.85)
4) 2 sweatshirts and 3 t-shirts cost $136. (Sweatshirts cost $41 and t-shirts cost $18)
5) 4 candy bars and 1 soda costs $12.35 (candy bars are $2.60 and sodas are $1.95)
6) 4 Sicilian and5 regular slices would cost $22.25 (Sicilian slice cost $2.75 and the regular slice cost $2.25)
7) 129 solid and 71 striped.
8) 48 adults and 72 child tickets
9) 407 quarters and 193 dimes
10) 22 introductory minutes and 13 additional minutes
11) 5 introductory minutes and 27 additional minutes
12) 190 quarters and 110 nickels